

# *Truss Vibration Control and Actuator Configuration with Concentrated Load at the End*

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**Abstract:** For the mathematical modeling of flexible structures, the finite element method is used to establish the vibration control and measurement equations of the truss structure. For the problem of flexible vibration suppression, the vibration actuator layout, vibration controller design are studied to construct a vibration control loop. In order to optimize the layout problem of the actuator, combined with the actual engineering requirements, an optimization index considering the controllability of the system state is proposed. And maneuvering simulation of vibration suppression was carried out for different installation methods of the truss.

## 1. Introduction





The truss structure has a broad application in the development of the aerospace industry, featuring light weight, weak structural damping, multiple nodes or special structural features, and have complex dynamic qualities such as low frequency, dense mode, and nonlinearity. The special outer space environment makes it prone to fail causing by large flexible deformations in the structure and uncontrollable large amplitude vibration. Appropriate methods are necessary for analyzing vibration characteristics and active control, which is theoretical and practical significant.

Active vibration control is generally the assembly of a certain type of actuator into the structure, through the control of the action of the actuator to generate control force, to achieve vibration suppression effect. The performance of the actuator plays a key role in the active vibration control. The development level of the actuator technology restricts the practical application of the active vibration control technology.

There are several common forms of truss structures used in space missions as shown in the form below: Stewart platforms, conventional diagonal trusses, polyhedral trusses, and annular tension trusses.

The Aerospace Structure Center of the Colorado University conducted dynamic and nano-vibration analysis studies on removable triangular tubular trusses<sup>[1]</sup>, and precision deployable optical trusses<sup>[2]</sup>. The Langley Research Center of NASA designed Phase II CSI Phase 0 Evolutionary Model and Tetrahedral truss platforms. Besides listed, there are also decahedron and dodecahedron forms. Literature<sup>[3]</sup> analyzed the support truss structure of tetrahedron, hexahedral and other mesh reflection antennas, similar to the study of CSI and PSR of NASA's LDR project.

Table 1. Common forms of truss structures.

Geometrtric type		Space load form description	Unit form
Stewart's Platform		Isolation between Spacecraft and High-Precision Large Deployable Space Antennas	
Diagonal Truss	Triangular/ Rectangular Truss	Outer Bearing Structure of Space Payload or Main Structure	
Polyhedral Truss	Tetrahedral/ Hexahedral/ Octahedral Truss	Supported Structures of Mesh Reflector Antennas; Parallel Robotic Manipulator	
Ring Tension Truss		Supported Structures of Mesh Reflector Antennas	

This paper includes two aspects of work: actuator installation position calculation and optimization and active vibration suppression using PD controller. In Section 4, simulations are provided under two working conditions of the horizontal and vertical placement with actuators selected in Section 2, and controller designed in Section 3. And finally conclusion is given in section 5.

## 2. Truss Model and Actuator Installing Position Calculation

Basic principles for deriving the dynamical equations of the flexible aerospace structure including Newton-Euler method, Lagrange analytical method and extreme principle based on of Gaussian principle. According to the model types established, there are distributed, lumped, hybrid coordinate, and finite element models<sup>[4]</sup>. The finite element model is the most widely used, which can establish dynamic model with high pointing accuracy<sup>[5]</sup>, and can be reduced in order or the spacecraft can be decomposed into a series of structural components represented by single finite element.

### 2.1 Truss Structure Mathematical Model

Assume that the number of quality nodes of the truss system is  $n_o$ , the number of actuators is  $n_a$ , the number of sensors is  $n_s$ , the finite element model of the entire truss system is,

$$M\ddot{\delta} + C\dot{\delta} + K\delta = f_d + Bf_a, \quad Q = C_a\ddot{\delta} + C_r\dot{\delta} + C_d\delta \quad (1)$$

Where,  $\delta \in R^{n \times 1}$  is nodes placement vector,  $M \in R^{n \times n}$ ,  $C \in R^{n \times n}$ ,  $K \in R^{n \times n}$  is system's total mass matrix, damping matrix and stiffness matrix.  $f_d \in R^{n \times 1}$  is the external disturbances to the truss.  $f_a \in R^{n \times 1}$  is the control force vector generated by actuators,  $Q \in R^{n \times 1}$  is the measurement vector,  $B \in R^{n \times n}$  is the actuator installation matrix.  $C_a, C_r, C_d \in R^{n \times n}$  is the installation matrix for acceleration, velocity and position sensors.

In practice, it is generally necessary to perform modal truncation on equation(1), and retain first low-order modes that have a great influence on performance indicators such as satellite attitude stability and pointing accuracy, while ignoring weak high-order modes. When modal truncation is performed, only the first  $n_a$  order of the modal coordinates  $\eta$  is retained.

A typical truss structure can be divided into longitudinal stringers as the main load-bearing unit to obtain the moment of inertia; the diagonal stringers to meet the requirements of shearing and torsion; also the horizontal stringers to maintain the section shape without bearing external load.

First it's necessary to determine the position coordinates of the centroid of the truss structure. The dynamic equations are established by the finite element method, the centroid coordinates of the flexible truss structure can be calculated according to the following formula.

$$M_x = \sum_{i=1}^n m_i \times x_i / \sum_{i=1}^n m_i, M_y = \sum_{i=1}^n m_i \times y_i / \sum_{i=1}^n m_i, M_z = \sum_{i=1}^n m_i \times z_i / \sum_{i=1}^n m_i \quad (2)$$

Where  $M_x, M_y, M_z$  represent the position coordinate of the centroid of the truss structure in the body frame.  $m_i$  represents the mass of finite element node  $i$ .  $x_i, y_i, z_i$  represent the position coordinate of the finite element nodes under body frame. After determining the centroid position of the truss structure, the moment of inertia of the truss structure can be calculated by the following formula,

$$J = \sum_{i=1}^n m_i \tilde{\mathbf{r}}_{bi}^T \tilde{\mathbf{r}}_{bi} \quad (3)$$

Where  $\tilde{\mathbf{r}}_{bi} = \begin{bmatrix} 0 & M_z - z_i & y_i - M_y \\ z_i - M_z & 0 & M_x - x_i \\ M_y - y_i & x_i - M_x & 0 \end{bmatrix}$

## 2.2 Actuators Installing Position Calculation and Optimization

The actuator of the vibration control system is called an actuator, piezoelectric active lever and a voice coil motor are commonly used. The actuator is mounted on the surface of the flexible structure to suppress the vibration.

### 2.2.1 Actuators installing position calculation

The installation of the actuator is represented by  $B$ , which contains position vector information of all the actuators. The control force vector provided by the actuator for each mass element node is

$$\mathbf{f}_c = \mathbf{B} \mathbf{f}_a = \begin{bmatrix} B_{11} & \cdots & B_{1n} \\ \vdots & \ddots & \vdots \\ B_{n1} & \cdots & B_{nn} \end{bmatrix} \begin{bmatrix} f_{a1} \\ \vdots \\ f_{an} \end{bmatrix} = \begin{bmatrix} f_{c1} \\ \vdots \\ f_{cn} \end{bmatrix} \quad (4)$$

According to equation(4), the control force  $f_{ci}$  acting on the  $i$ -th node can be specifically written as

$$f_{ci} = B_{i1} f_{a1} + B_{i2} f_{a2} + \cdots + B_{in} f_{an} \quad (5)$$

The  $i$ -th row of  $B$  in turn represents the influence of force  $n_c$ -th actuator have on the  $i$ -th node. Then construct a corresponding matrix of installing positions based on the actual position of actuators.

Assume that the actuator can be bridged across two adjacent nodes. As shown in Fig. 2, if the first actuator is installed along the direction of the finite element node  $1 \rightarrow 2$ ,  $F_c$  is the magnitude of the force generated by the actuator,  $\alpha, \beta, \gamma$  is the angle between the actuator and the three axes of the body frame, the actuator. Then the direction cosine of the actuator is  $\cos \alpha, \cos \beta, \cos \gamma$  respectively.

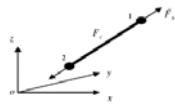


FIGURE 1. Node force analysis diagram

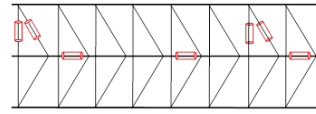


FIGURE 2. Actuator installation location diagram

The force analysis of the node 1 can be obtained as  $\vec{F}_1 = [F_c \cos \alpha \ F_c \cos \beta \ F_c \cos \gamma]^T$ , the force analysis of the node 2 is  $\vec{F}_2 = [-F_c \cos \alpha \ -F_c \cos \beta \ -F_c \cos \gamma]^T$ .

According to the force analysis, the 1-th row of installation matrix  $B$  is  $B_1 = [(\cos \alpha \ \cos \beta \ \cos \gamma)^T \ 0 \ \dots \ 0]$ , the 2-nd row of installation matrix  $B$  is  $B_2 = [(-\cos \alpha \ -\cos \beta \ -\cos \gamma)^T \ 0 \ \dots \ 0]$

The rest actuators are analyzed in turn and the corresponding actuator direction cosine is written into the installing position matrix to establish a complete actuator installation position matrix.

### 2.2.2 Actuators installing position optimization

In order to design the installing position, first select some on the structure as the optional installing positions, and then compare these optional ones according to the optimization indicators, and finally obtain the best scheme. As can be seen in fig.3, the truss structure can be approximated as a repetition of four units, the central steel rod is added to the first unit, and the triangular steel is fixed at the right end of the last unit.

Keep the actuator bridges across two nodes, some adjacent nodes are periodically selected as optional installing locations. A total of 24 positions are selected among the four units for the actuator, and the selectable number is shown in fig.3. Calculate the position where the deformation variable is the largest as the installation position of the actuator.

According to equation (1), the modal control force vector of the truss structure vibration is  $u = U^T B f_a = [u_1 \ \dots \ u_n]^T$  (6). Among them, the modal control force of the first-order vibration mode can be written as:  $u_1 = U_1^T B f_a$  (7)

The type of actuator depends primarily on the magnitude of the actuator output control force during the vibration control process. The magnitude of the output force of the actuator is mainly determined by the following three factors<sup>[6]</sup>: 1) The vibration amplitude of the flexible truss structure; 2) The value of  $U_1^T B$  at the actuator mounting position; 3) The number of actuators installed.

Take the suboptimal scheme and install multiple actuators for the first-order mode, the magnitude of the output force can be reduced. However the multiple installations reduce the controllability to a certain extent and increases the system energy consumption. The positions of the actuator are the optimal one to ensure the highest efficiency.

## 3. Pd Central Body Attitude Maneuvering Pid Step-By-Step Saturation Controller

For the attitude dynamics equation of truss structure(1), considering the control torque and angular velocity constraints, the following PID step-by-step saturation controller is designed

$$\tau = -J \left\{ 2p \text{sat}_{\tau} \left( e + \frac{1}{T} \int e \right) + d\omega \right\} \quad (8)$$

where  $p, d$  are controller parameters,  $T$  is the integration time constant. Saturation function  $\text{sat}(\cdot)$  is used to realize the limit of each elements, with the max value is  $L_i$  ( $i = 1, 2, 3$ ) define as follow,

$$L_i = \frac{d}{2p} \{ |\omega_i|_{\max} \} \quad (9)$$

Where  $|\omega_i|_{\max}$  is the maximum angular velocity allowed for each axis of the spacecraft. Controller (8) make the actuators able to exert the maximum desired torque direction with proper parameter selection. During the maneuvering process, the angular velocity exhibits an acceleration-synchronization-deceleration variation law.

#### 4. Numerical Simulation

The dimensions of the truss structure are shown in the figure. The root is solid steel rod fixed with the foundation; the section unit is a triangular frame with a unilateral length of 543.6 mm; the load at the end is reduced to an overall triangular steel structure with a mass of 83.86 kg, constant force spring is used to trim the gravity.

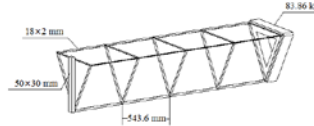


FIGURE 3. Truss structure diagram

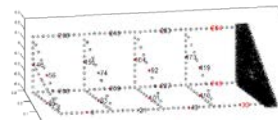


FIGURE 4. Actuator installing position chosen

The payload supported by the space truss is reduced to a centralized mass which does not change the inherent characteristics and can only affect the overall response of the system. Therefore, the concentrated mass is attributed to the load parameter as an external input factor, and the optional installing position is not considered.

Using the given finite element data, according to equations (2) and (3), the centroid coordinates and moment of inertia of the truss structure are  $M_x = 1.90 \text{ m}$ ,  $M_y = -4.6e^{-3} \text{ m}$ ,  $M_z = 0 \text{ m}$ . Then calculating the first three-order translational coupling coefficient using Broti\_MARTIX, and the result is shown in the chart below.

Table 2. First three-order translational coupling coefficient (relative to the installed coordinate system).

Modal	x	y	z
1	0.000	0	-0.4353
2	0.000	0	-9.2764
3	0.527	-9.2756	0

The vibration deformation of the first 3 orders is as shown in the fig.5 without applying control force.

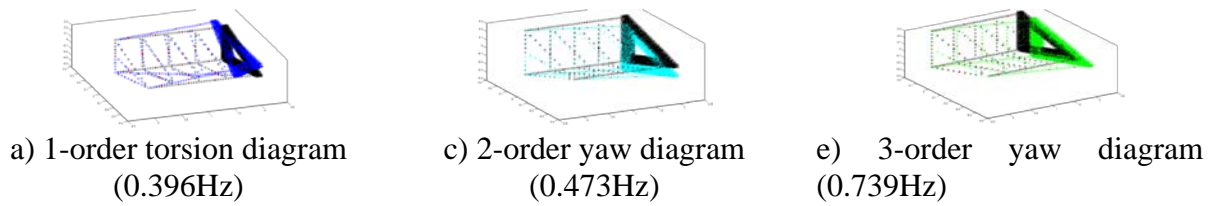


FIGURE 5. 1~3 order vibration without control force

Set appropriate flexible truss damping coefficient [0.01, 0.01, 0.01]. Although the system response gradually converges after a sufficiently long time, the truss will eventually have dense, low-frequency small-amplitude vibrations, which will seriously affect the stability of the overall structure.

For equation (7), the installing position at which the actuator produces maximum modal control is the position of  $U_1^T B$  in all selectable positions. Calculate the  $B$  matrix of 24 optional installation positions, arrange the obtained results.

For the first three modes of modal synthesis, choose to install the actuator at three node positions 254, 218, and 33, and similarly for other high-order vibrations. Actuators No.218 and No.254 are structurally symmetrical and are also optimally selectable positions for the first and second-order mode, which means that the actuators can be installed at both nodes simultaneously to provide effective vibration control for first-order torsion and second-order yaw.

The first two modes converge faster, and the third one has a small oscillation adding appropriate PD controller. The simulation results of vibration suppression under the horizontal and vertical placement are given below. The CMG is adjusted to avoid the system divergence causing by the interference torque generated by the flexible vibration.



FIGURE 6. Horizontal and vertical placement of truss structure

In the case where a vibration control actuator is installed on the surface of the truss structure, the controller (8) is used to simulate the posture maneuvering when the truss structure. In the case where the vibration control actuator is not installed on the surface of the truss structure, the controller (9) is used to perform the attitude maneuver simulation of the truss structure.



FIGURE 7. Horizontal and vertical placement maneuver - body attitude curve (no vibration control)

Fig.7 shows that in the case where the overall structure is free to vibrate, the overall structure can still perform the attitude maneuvering task. However, due to the existence of flexible vibration, the angular velocity curve fluctuates greatly during the acceleration and deceleration of the body, especially during the end of deceleration. During the uniform motion, the angular velocity error is difficult to meet the high accuracy requirements.

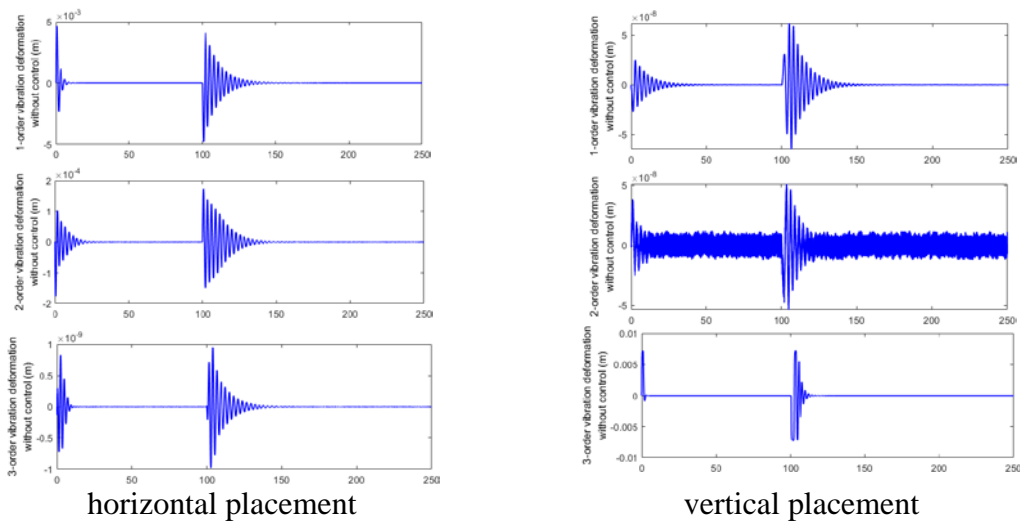


FIGURE 8. Structural end vibration displacement curve (no vibration control)

Fig.8 shows that the end of the structure deforms greatly during the attitude maneuvering process, and there is a period of vibration, which is difficult to meet the mission requirements. In the case of the free vibration of the overall structure, the body can complete the slow attitude maneuver, but the attitude determination accuracy, stability and motion quality are difficult to meet the actual task requirements.

By analyzing the vibration coupling coefficient in Table 2, it can be known that when the truss structure is placed horizontally, when the body structure is maneuvering, the third-order vibration is most likely to be excited due to its large first-order coupling coefficient. According to the formula (2), the influence of the third-order vibration on the control accuracy is also the most significant. Therefore, the vibration controller is designed for the first, second, and third-order vibrations respectively, which can effectively improve the control accuracy.

Expected attitude angle is  $[50\ 0\ 0]^\circ$ , maximum allowable angular velocity is  $0.05^\circ/s$ . Attitude controller parameters are  $p=9.54$ ,  $d=5.5$ ,  $T=10$ .

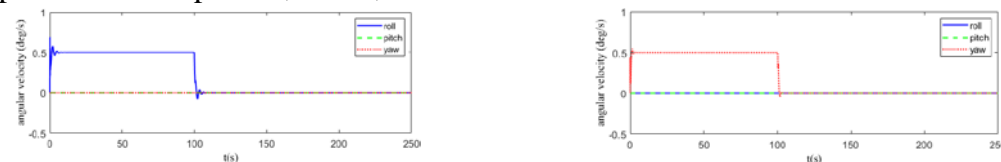


FIGURE 9. Horizontal placement maneuver - body attitude curve (with vibration control)

It can be seen from Fig. 9 that after applying the first three-order active vibration control to the truss structure, the attitude control accuracy is obviously improved, and the center body completes the attitude maneuvering task in about 100s, the attitude pointing accuracy is better than  $0.001^\circ$ , and the attitude stability is better than  $0.0001^\circ/s$ . The angular velocity of motion is better than  $0.0001^\circ/s$  during the whole motion.



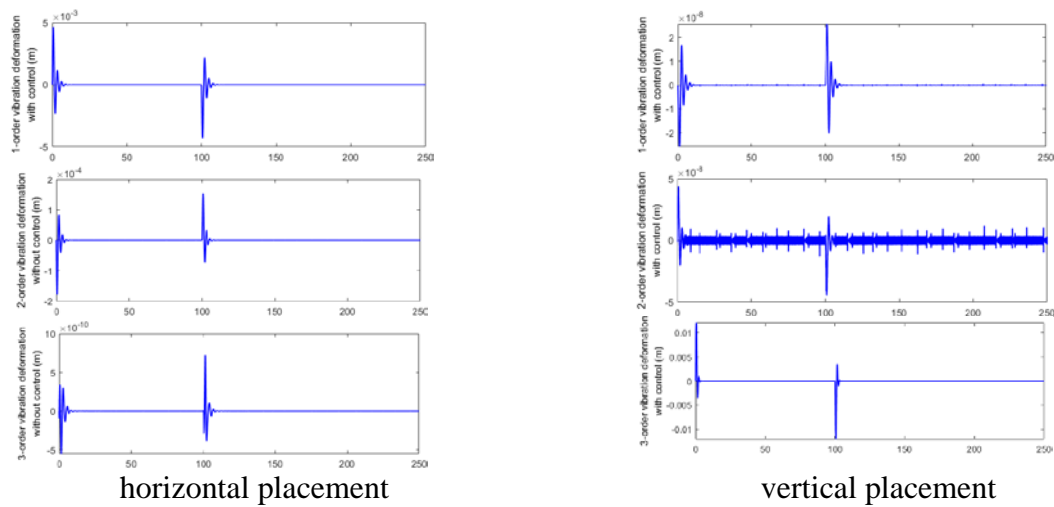


FIGURE 10. Structural end vibration displacement curve (with vibration control)

Comparing Fig. 10 with Fig. 8, it can be seen that the first, second and third order vibrations of the truss structure are effectively suppressed.

## 5. Conclusion

Comparing the maneuver simulation results of horizontal placement and vertical placement, it is found that the truss structure shape variable is small in the vertical placement. For the first three-order vibration, the controlled maneuver simulation after the actuator is installed, the truss structure shape variable is smaller in the vertical placement, and the actuator output torque is smaller. Therefore, it is reasonable to consider the truss structure in the form of vertical placement.

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